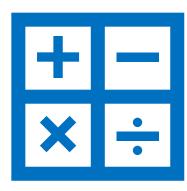
Calculation Policy

Mathematics

'Spirituality is the bitter-sweet yearning for beauty, truth, love and wonder beyond ourselves. It is a longing we pursue together and a treasure we glimpse in ourselves and one another and seek beyond us into eternity. It is life in all its fullness.'



Nebula Spirituality Statement









This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy.

Recommended practice delivering a mastery approach

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondary school.

Evidence repeatedly shows that mixed ability seating increases less confident pupils' perception of mathematical capability, which impacts positively upon outcomes. While not a school policy, it is recommended to avoid ability groups. This presents a challenge in ensuring the more confident mathematicians are being extended. An extension tasks to deepen understanding is the most simplistic way around this.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and thinkers. They can also provide a common language with which to communicate cognitive models for abstract ideas. Drury, H. (2015) Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. Jean Piaget's (1951)

Real things and structured images enables children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it's important to move between all modes to allow children to make connections. Morgan, D. (2016)

The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

Reception Addition

Objective/Strategy	Concrete	Pictorial	Abstract
Find 'one more than' a number	Use cubes to count out a given number. Find and add one more.	Use pictures to find one more by counting on. Use a number line to support with writing the answers.	4+1= $0+1=$ $2+1=$ $5+1=$ Use cubes to count on as in the previous examples to support moving into the abstract.
To explore the composition of numbers to 10. Automatically recall number bonds for numbers 0–5 and some to 10.	Use two colours of cubes to create a range of representations of a given number. Use Numicon to 'mirror' and match shapes to make a given number.	Put 5 cupcakes on two plates. 2 and 3 make 5 5 Whole 2 Part 3 Part Use pictures to represent numbers in different ways.	Emphasis should be on the introduction and building up of subject-specific vocabulary through practical work.

Year 1 Addition

Objective/Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	Use part, part whole model. Use cubes to add two numbers together as a group or in a bar	Use pictures to add two numbers together as a group or in a bar. 8 1 Solution 1 Solution 1 Solution 2 Solutio	8 = 5 + 3 5 + 3 = 8 Use the part, part, whole diagram as shown above to move into the abstract. Include missing number questions to support varied fluency: 8 = ? + 3 5 + ? = 8
Starting at the bigger number and counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10 This is an essential skill for column addition later.	6 + 5 = 11	3 + 9 = Use pictures or a number line. Regroup	7 + 4= 11 If I am at seven, how many more do I
	Start with the bigger number and use the smaller number to make 10. Use ten frames.	or partition the smaller number using the part, part whole model to make 10.	need to make 10? How many more do I add on now?

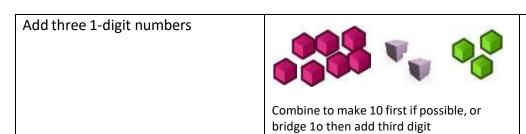
Year 1 Addition continued... **Objective/Strategy** Concrete **Pictorial Abstract** Represent & use number bonds and Include missing number questions: related subtraction facts within 20 8 = ? + 3 5 + ? = 8 2 more than 5. Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' 5 + 2 ='8 is 3 more than 5.'

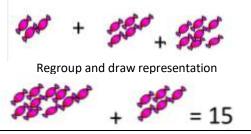
Year 2
Addition

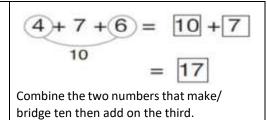
	Add	ition	
Objective/Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20 Model using dienes and bead strings	3 fens + 5 tens =	20 + 30 = 50 70 = 50 + 20 40 + \square = 60
Use known number facts Part, part whole	Children explore ways of making numbers within 20	20	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtraction and use this to check calculations.
Using known facts		 ∴ + ∴ = ∴ (+ = 	3 + 4 = 7 leads to 30 + 40 = 70 leads to 300 + 400 = 700
Bar model	3 + 4 = 7	7 + 3 = 10	23 25 ? 23 + 25 = 48

Year 2 Addition continued...

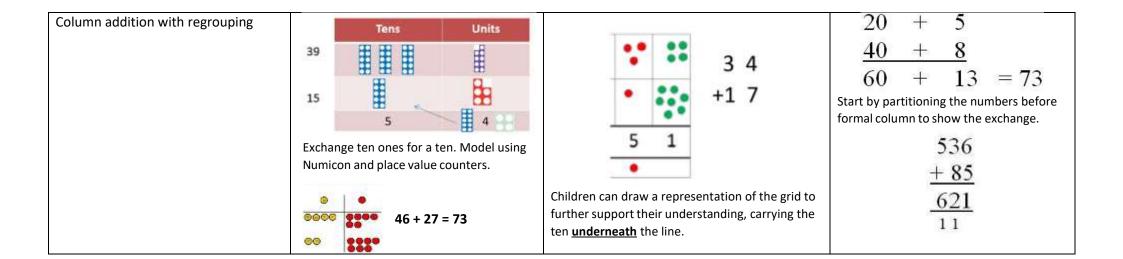
Objective/Strategy	Concrete	Pictorial	Abstract
Add a two-digit number and ones	17 + 5 = 22 Use ten frame to make 'magic ten' Children explore the pattern: 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 17 + 5 = 22 Use part part whole and number line to 16 + 7 16 + 7	Explore related facts: 17 + 5 = 22 5 + 17 = 22 22-17 = 5 22-5 = 17 Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.
Add a 2-digit number and tens	25 + 10 = 35 Explore that the ones digit does not change.	27 + 30 +10 +10 +10 27 37 47 57	27 + 10 = 37 27 + 20 = 47 27 + \square = 57
Add two 2-digit numbers	Model using dienes, place value counters and Numicon	Use number line and bridge ten using part whole if necessary.	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 = 12 60 + 12 = 72







Concrete	Pictorial	Abstract
Dienes or Numicon		
Add together the ones first, then the tens.	tens ones	2 2 3
15 Units 45		+ 1 1 4
7 9		3 3 7
© © © Cakalettons 21 + 42 = 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	Children move to drawing the counters using a 'tens and ones' frame.	Add the ones first, then the tens, then the hundreds.
	Concrete Dienes or Numicon Add together the ones first, then the tens.	Add together the ones first, then the tens. Children move to drawing the counters using a 'tens and ones' frame.



Year 3 Addition continued			
Objective/Strategy	Concrete	Pictorial	Abstract
Estimate the answers to questions and use inverse operations to check answers	***************************************	Use number lines to illustrate estimation	Building up known facts and using them to illustrate the inverse and to check answers:
	Estimating 98 + 17 = ? 100 + 20 = 120	90 100	98 + 18 = 116

Years 4, 5 and 6
Addition

	A	adition	
Objective/Strategy	Concrete	Pictorial	Abstract
Year 4/5/6 Estimate and use inverse operations to check answers to a calculation		As per Year 3	
Year 4 add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.	7 1 5 1 Draw representations using place value grid.	3517 + 396 3913 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Year 5 add numbers with more than 4 digits add decimals with 2 decimal places, including money	As per Year 4 Tens ones tenths hundredths	2.37 + 81.79 +ens ones +ents hundreates 000 0000 0 000000 000000 0 000000 000000	72.8 +54.6 127.4 1 1 € 2 3 · 5 9 + £ 7 · 5 5 € 3 · 4
Year 6 add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points	As per Year 5	As per Year 5	Insert zeros for place holders. 8 1,05 9 3,66 8 15,30 1 + 20,55 1 1 20,579 1 1 3 00 + 1 3 00 93 5 1 1

Reception Subtraction

Subtraction			
Objective/Strategy	Concrete	Pictorial	Abstract
Find 'one less than' a number		Ó Ó X	
			Use concrete and pictorial examples to support moving into the abstract.
	Use cubes to count out a given number. 'Take one away' and count to find 'how many left'.	404040-40=	6-1=
		Use pictures to 'take one away'. Count to find out 'how many left'.	
To explore the composition of numbers to 10.	feely bag		Emphasis should be on the introduction and building up of subject-specific vocabulary through practical work.
Automatically recall number bonds for numbers 0–5 and some to 10.	Use a feely bag to 'hide' a number of cubes. "I have 5 cubes altogether. I can see 2 cubes. How many have I hidden in my bag?"		
	Throw beanbags into a hoop. How		
	many went in? How many are left outside?		

Year 1 Subtraction

Objective/Strategy	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. $4-2=2$ $6-4=2$	Cross out drawn objects to show what has been taken away. $ \begin{array}{cccccccccccccccccccccccccccccccccc$	7—4 = 3 16—9 = 7
Counting back	Move objects away from the group, counting backwards. Move the beads along the bead string as you count backwards.	5 - 3 = 2 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts 7 'Seven is 3 more than four' 4 'I am 2 years older than my sister' 5 Fench Lay objects to represent bar model	Count on using a number line to find the difference. *6 0 1 2 3 4 5 6 7 8 9 10 11 12	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?

Year 1
Subtraction continued

Objective/Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Include subtracting zero Part, Part Whole model	Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? $10-6=4$	Lisa nistarial representations to show the	Move to using numbers within the part whole model. 5 12 7 Include missing number problems: 12
	10-0-4	Use pictorial representations to show the part.	- ? = 5 7 = 12 - ?
Make 10	Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13 - 7 = 6 3 4 5 1 2 3 4 5 6 7 8 8 10 11 12 13 14 15 16 17 18 19 30 13-7 Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model Including the inverse operations.	5-2=3	3333333 3 3	8 2 10 = 8 + 2 10 = 2 + 8 10-2 = 8 10-8 = 2

Year 2
Subtraction

Subtraction				
Objective/Strategy	Concrete	Pictorial	Abstract	
Regroup a ten into ten ones	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	70000 = 4 70000 = 20	20—4 = 16	
Partitioning to subtract without regrouping 'Friendly numbers'	Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.	43—21 = 22	
'Make ten' strategy Progression should be crossing one ten, crossing more than one ten, crossing the hundreds	34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	Use a number line to count on to next ten and then the rest.	93—76 = 17	

Year 3 Subtraction

Subtraction			
Objective/Strategy	Concrete	Pictorial	Abstract
Subtract numbers mentally, including:	- The same of the	90 100	Vary the position of the answer and question.
three-digit number + ones	Observed #111-		Expose children to missing number
three-digit number + tens			questions and vary the missing part of the calculation.
three-digit number + hundreds			678 = ? – 1 688 – 10 = ?
			678 = ? – 100
Column subtraction without regrouping	111 22	Calculations	47-24= ²³
'Friendly numbers'		5/2	20+3 20+3
Friendly numbers	47—32	32	Intermediate step may be needed to
	Use Base 10 or Numicon to model	Draw representations to support understanding	lead to clear subtraction understanding
Column subtraction with regrouping	Tens Units	Tens lones	836-254-582 Begin by partitioning into PV columns
	Begin with base 10 or Numicon. Move to PV counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange	Children may draw base ten or PV counters and cross off	7 28 - 582 = 146 Then move on to formal method

Years 4, 5 and 6 Subtraction

Subtraction				
Objective/Strategy	Concrete	Pictorial	Abstract	
Year 4 subtract tens and ones with up to 4 digits introduce decimal subtraction through context of money	234 - 179 O O O O O O O O O O O O O O O O O O O	Children to draw PV counters and show their exchange—see Year 3	2 x 5 4 - 1 5 6 2 1 1 9 2 Use the phrase 'take and make' for exchange	
Year 5 subtract with at least 4 digits, including money and measures subtract with decimal values, including mixtures of integers and decimals and aligning the decimal point (up to 3 decimal places)	As per Year 4	Children to draw PV counters and show their exchange—see Year 3	"%" 10 % 6 - 2 2 8 2 8,9 2 8 Use zeros 7 8 6 9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	
Year 6 Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal places)	As per Year 4	Children to draw PV counters and show their exchange—see Year 3	**************************************	

Reception
Multiplication

Multiplication				
Objective/Strategy	Concrete	Pictorial	Abstract	
Subitise			Emphasis should be on the introduction and building up of subject-specific vocabulary through	
Compare numbers	Roll a pair of dice to find doubles	1 + 1 = 2 + 2 = 3 + 3 =	practical work.	
		Use pictures to find doubles by drawing the same again		
	Match pairs of Dominoes to find			
	doubles			

Year 1			
Multiplication			
Objective/Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling	Draw pictures to show how to double numbers Double 4 is 8	Partition a number and then double each part before recombining it back together 16 10 6
	double 4 is 8 4×2=8 + = =		20 12 20 + 12 = 32
Counting in multiples (2s, 5s, 10s)	Count the groups as children are skip counting, children may use their fingers as they are skip counting.	Children make representations to show counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	Use manipulatives to create equal groups.	Draw to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

Year 1 Multiplication continued...

iviuitipiication continued				
Objective/Strategy	Concrete	Pictorial	Abstract	
Repeated addition	Use different objects to add equal groups	Use pictorial including number lines to solve problems prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 3+3+3+3+3 = 15	Write addition sentences to describe objects and pictures	
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.	Draw representations of arrays to show understanding	3 x 2 = 6 2 x 5 = 10	

Year 2

Multiplication

Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times tables.			
Objective/Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters. 40 +12 = 52	Draw pictures and representations to show how to double numbers	As per Year 1
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 5+5+5+5+5+5+5+5=40 Use bar models.	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 3 3 3 3 3	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 4 × 3 =

Year 2
Multiplication continued

Multiplication continued				
Objective/Strategy	Concrete	Pictorial	Abstract	
Multiplication is commutative	Create arrays using counters, cubes or Numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.	Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15 12 = 3 x 4 12 = 4 x 3	
Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other.		X =	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.	

Year 3

Multiplication

Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables.

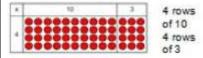
Objective/Strategy

Grid method, progressing to the formal method

Multiply 2-digit numbers by 1-digit numbers

Concrete

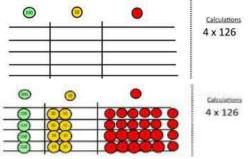
Show the links with arrays to first introduce the grid method.



Move onto base ten to move towards a more compact method.



Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows



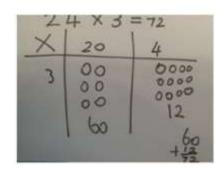
Fill each row with 126.

Add up each column, starting with the ones making any exchanges needed Then you have your answer.

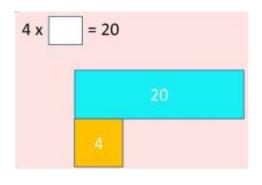
Pictorial
Children can represent their work with place

value counters in a way that they understand.
They can draw the counters using colours to

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar models are used to explore missing numbers



Start with multiplying by one-digit

Abstract

numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

$$210 + 35 = 245$$

Move forward to the formal written method:

Years 4, 5 and 6 Multiplication

	iviui	tiplication	
Objective/Strategy	Concrete	Pictorial	Abstract
Grid method to recap from Year 3 for 2-digit x 1-digit Move to multiplying 3-digit numbers by 1-digit Year 4 expectation	As per Year 3	As per Year 3	As per Year 3
Column Multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This is initially done where there is no regrouping, such as: 321 x 2 = 642 It is important at this stage that they always multiply the ones first. The corresponding expanded multiplication is modelled alongside.	x 300 20 7 4 1200 80 28	327 x 4

Years 4, 5 and 6 Multiplication

Wattiplication					
Objective/Strategy	Concrete	Pictorial		Α	bstract
Column multiplication	Manipulatives may still be used with	11-2			
(Long multiplication)	the corresponding long multiplication modelled alongside.	10 8	1	8	
,	3		× 1	3	
		10 100 80	5	4	i .
		3 30 24	1 8		
		3 30 25	2 3	4	
		102			
		Continue to use bar modelling to support pro	oblem solvin	S	•
				1 d	1 1 1
Multiplying decimals up to 2 decimal places by a single digit.			belongs in t	ne Units	et the single digit column. Line up the e question and the
			3 .	1	9
			x 8		
			25	. 5	2
				7	

Reception
Division

Pictorial ere are 6 cakes. Can you share them? Try saying "one for me, one for you" A A A A A A A A A A A A A A A A A A A	Abstract Emphasis should be on the introduction and building up of subject-specific vocabulary through practical work.
Try saying "one for me, one for you"	introduction and building up of subject-specific vocabulary through
Use pictures to share	

		Year 1	
		Division	
Objective/Strategy	Concrete	Pictorial	Abstract
Division as sharing (Use Gordon ITPs for modelling)		Children use pictures or shapes to share quantities:	12 shared between 3 is 4. 12 ÷ 3 = 4
	I have 10 cubes; can you share them equally in 2 groups?	8 shared between 2 is 4. Sharing:	
	10	Children use bar modelling to show and support understanding. 12 shared between 3 is 4 Children use bar modelling to show and support understanding. 12 ÷ 4 = 3	
Division as grouping	Divide quantities into equal groups Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping 12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 20 20 21 20 20 20 20 20 20 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Year 2 **Division Objective/Strategy Concrete Pictorial Abstract** Use cubes, counters, objects or place value Continue to use bar modelling to aid solving How many groups of 6 in 24? Division as grouping counters to aid understanding. division problems. $24 \div 6 = 4$ 20 24 divided into groups of 6 = 4 $20 \div 5 = ?$ 5 x ? = 20 $96 \div 3 = 32$ Division with arrays Draw an array and use lines to split the array into Find the inverse of multiplication and groups to make multiplication and division division sentences by creating eight sentences: linking number sentences. $7 \times 4 = 28$ $28 = 7 \times 4$ $4 \times 7 = 28$ $28 = 4 \times 7$ $28 \div 7 = 4$ $4 = 28 \div 7$ Link division to multiplication by creating an $7 = 28 \div 4$ $28 \div 4 = 7$ array and thinking about the number sentences that can be created. $15 \div 3 = 5$ $5 \times 3 = 15$ 3 x 5 = 15 $15 \div 5 = 3$ $15 \div 3 = 5$

Year 3 **Division** (Greater Depth Y2) **Objective/Strategy** Concrete **Pictorial Abstract** Division with Divide objects between groups and see how Jump forward in equal jumps on a number Complete written divisions and show the remainders much is left over: line then see how many more you need to remainder using r. jump to find a remainder: 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder Draw dots and group them to divide an amount and clearly show a remainder: Use bar models to show division with remainders: $14 \div 3 = 4$ with 2 left over 37 96 ÷ 3 10 10 10 Calculations 42 + 3 Use place value counters to divide using the bus stop method alongside: 42 ÷ 3= We look how much in 1 group, so the answer is 14

Years 4, 5 and 6 **Division Objective/Strategy** Concrete **Pictorial Abstract** Begin with division that divide equally with Divide at least 3-digit numbers by a 1-Students can continue to use drawn diagrams no remainder with dots or circles to help them divide numbers digit number into equal groups **Short Division** Move onto division with a remainder Encourage them to move towards counting in As per Year 3 multiples to divide more efficiently $96 \div 3 =$ Finally move into decimal places to divide the total accurately 30 + 2 = 32

Year 6 Division

Division			
Objective/Strategy	Abstract		
ong division	Long division: a step-by-step guide Long division is set out in the following way. 15 3640 15 into 3 doesn't go, so look at the next digit. 15 3640 15 goes into 36 two times, so put a 2 above the 6. 15 x 2 = 30 Take that 30 away from the 36 to get your remainder. 36 - 30 = 6 Next, carry the 4 down to make 64. 15 goes into 64 four times, so put a 4 above the 4. 15 x 4 = 60 Take 60 from the 64 to get your remainder.	34 15 34 1598 -136 23	At the side of the calculation, do jottings for multiples of 34. 34 doesn't go into 15, so look for the closest multiple to 159. 4 x 34 is 136 so subtract this from 159, leaving 23. Move the 8 that is left in the dividend down to make 238 left. Use the jottings created already to fin the closest multiple of 34 to 238, in this
	4 64 - 60 = 4 242 15 3640 - 30	238	case it is 7 x 238, which makes a quotient of 47. If there is a remainder this can be recorded as a remainder, or fraction, or the method can be extended to incorporate decimals as necessary.