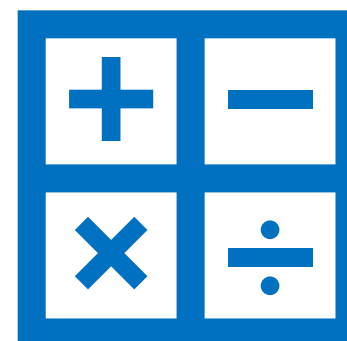


# Calculation Policy

## Mathematics

*‘Spirituality is the bitter-sweet yearning for beauty, truth, love and wonder beyond ourselves. It is a longing we pursue together and a treasure we glimpse in ourselves and one another and seek beyond us into eternity. It is life in all its fullness.’*



### Nebula Spirituality Statement



*This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy.*

## **Recommended practice delivering a mastery approach**

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondary school.

Evidence repeatedly shows that mixed ability seating increases less confident pupils' perception of mathematical capability, which impacts positively upon outcomes. While not a school policy, it is recommended to avoid ability groups. This presents a challenge in ensuring the more confident mathematicians are being extended. An extension tasks to deepen understanding is the most simplistic way around this.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

*Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and thinkers. They can also provide a common language with which to communicate cognitive models for abstract ideas. Drury, H. (2015)*


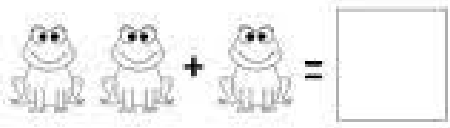
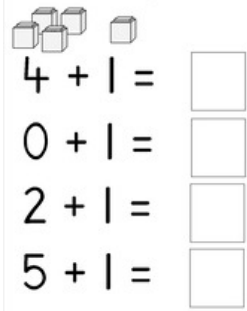
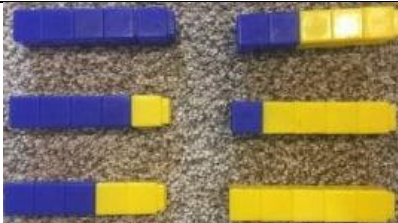
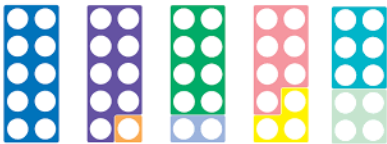

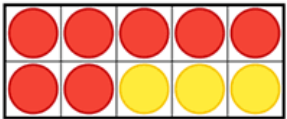
*Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. Jean Piaget's (1951)*

*Real things and structured images enables children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it's important to move between all modes to allow children to make connections. Morgan, D. (2016)*

The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

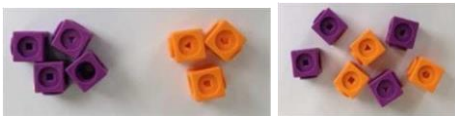

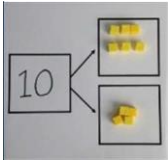

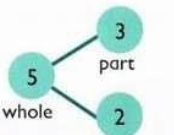

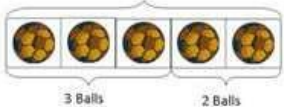


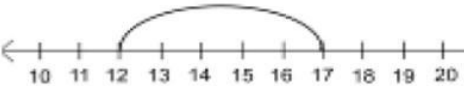


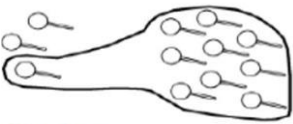

# Reception

## Addition

Objective/Strategy	Concrete	Pictorial	Abstract
Find 'one more than' a number	 <p>Use cubes to count out a given number. Find and add one more.</p>	 <p>Use pictures to find one more by counting on. Use a number line to support with writing the answers.</p>	 <p>Use cubes to count on as in the previous examples to support moving into the abstract.</p>
<p>To explore the composition of numbers to 10.</p> <p>Automatically recall number bonds for numbers 0–5 and some to 10.</p>	 <p>Use two colours of cubes to create a range of representations of a given number.</p>  <p>Use Numicon to 'mirror' and match shapes to make a given number.</p>	<p>Put 5 cupcakes on two plates.</p>  <p>2 and 3 make 5</p> <p>5 Whole 2 Part 3 Part</p>  <p>Use pictures to represent numbers in different ways.</p>	<p><b>Emphasis should be on the introduction and building up of subject-specific vocabulary through practical work.</b></p>


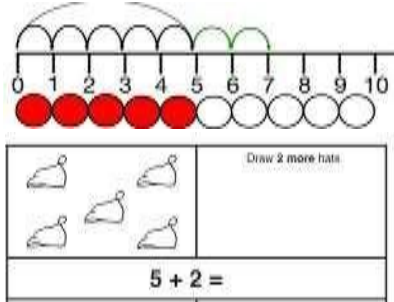
# Year 1

## Addition

Objective/Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 <p>Use part, part whole model. Use cubes to add two numbers together as a group or in a bar</p>  	<p>Use pictures to add two numbers together as a group or in a bar.</p> <p>8      1</p>    	<p><b><math>8 = 5 + 3</math></b> <b><math>5 + 3 = 8</math></b></p>  <p>Use the part, part, whole diagram as shown above to move into the abstract.</p> <p>Include missing number questions to support varied fluency:</p> <p><b><math>8 = ? + 3</math></b> <b><math>5 + ? = 8</math></b></p>
Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	 <p><b><math>12 + 5 = 17</math></b> Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p><b><math>5 + 12 = 17</math></b></p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
Regrouping to make 10  This is an essential skill for column addition later.	 <p><b><math>6 + 5 = 11</math></b></p>  <p>Start with the bigger number and use the smaller number to make 10. Use ten frames.</p>	 <p><b><math>3 + 9 =</math></b></p> <p>Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10.</p> <p><b><math>9 + 5 = 14</math></b></p> 	<p><b><math>7 + 4 = 11</math></b></p> <p>If I am at seven, how many more do I need to make 10? How many more do I add on now?</p>


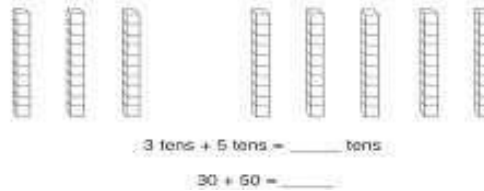
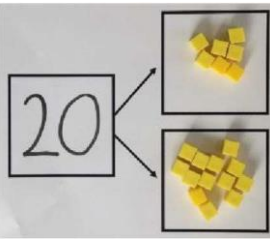
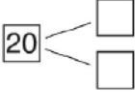
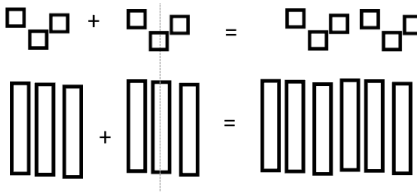
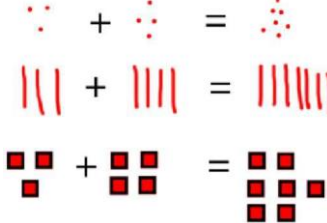


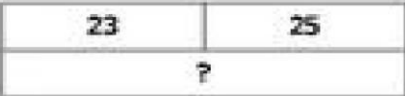
# Year 1

## Addition continued...

Objective/Strategy	Concrete	Pictorial	Abstract
Represent & use number bonds and related subtraction facts within 20	 <p>2 more than 5.</p>		<p>Include missing number questions:</p> <p><b><math>8 = ? + 3</math></b></p> <p><b><math>5 + ? = 8</math></b></p> <p>Emphasis should be on the language '1 more than 5 is equal to 6.'</p> <p>'2 more than 5 is 7.'</p> <p>'8 is 3 more than 5.'</p>

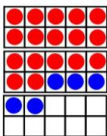
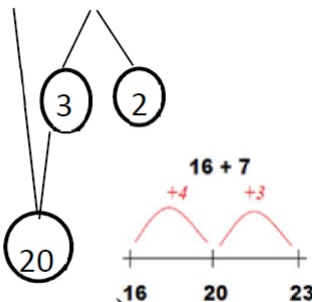
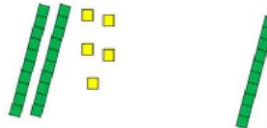
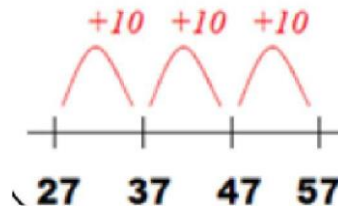
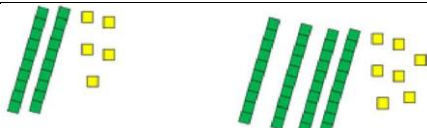
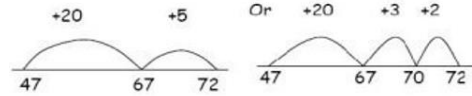
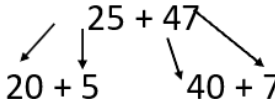
## Year 2


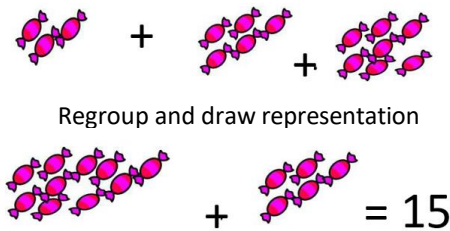
### Addition

Objective/Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$ $30 + 50 = \underline{\hspace{2cm}}$	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts  Part, part whole	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtraction and use this to check calculations. $\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts		 Children draw representations of H, T and O	$3 + 4 = 7$ leads to $30 + 40 = 70$ leads to $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$

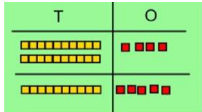
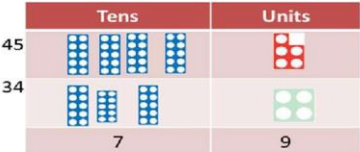
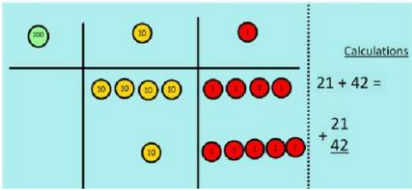
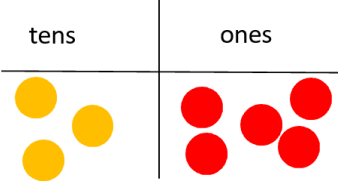
# Year 2

## Addition continued...

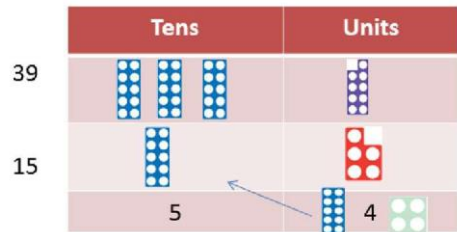
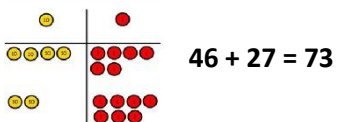
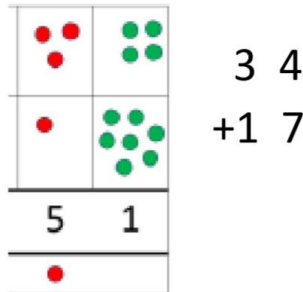
Objective/Strategy	Concrete	Pictorial	Abstract				
Add a two-digit number and ones	<div></div> <div><math>17 + 5 = 22</math></div> <div>Use ten frame to make 'magic ten'</div> <div>Children explore the pattern:</div> <div><math>17 + 5 = 22</math></div> <div><math>27 + 5 = 32</math></div>	<div><math>17 + 5 = 22</math></div> <div>Use part part whole and number line to model.</div> <div></div>	<div><math>17 + 5 = 22</math></div> <div>Explore related facts:</div> <div><table><tr><td><math>17 + 5 = 22</math></td><td><math>5 + 17 = 22</math></td></tr><tr><td><math>22 - 17 = 5</math></td><td><math>22 - 5 = 17</math></td></tr></table></div> <div>Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.</div>	$17 + 5 = 22$	$5 + 17 = 22$	$22 - 17 = 5$	$22 - 5 = 17$
$17 + 5 = 22$	$5 + 17 = 22$						
$22 - 17 = 5$	$22 - 5 = 17$						
Add a 2-digit number and tens	<div></div> <div><math>25 + 10 = 35</math></div> <div>Explore that the ones digit does not change.</div>	<div><math>27 + 30</math></div> <div></div>	<div><math>27 + 10 = 37</math></div> <div><math>27 + 20 = 47</math></div> <div><math>27 + \square = 57</math></div>				
Add two 2-digit numbers	<div></div> <div>Model using dienes, place value counters and Numicon</div>	<div></div> <div>Use number line and bridge ten using part whole if necessary.</div>	<div></div> <div><math>20 + 40 = 60</math></div> <div><math>5 + 7 = 12</math></div> <div><math>60 + 12 = 72</math></div>				

Add three 1-digit numbers	 <p>Combine to make 10 first if possible, or bridge 10 then add third digit</p>	 <p>Regroup and draw representation</p>	$\begin{array}{r} 4 + 7 + 6 = 10 + 7 \\ \quad \quad \quad 10 \\ = 17 \end{array}$ <p>Combine the two numbers that make/bridge ten then add on the third.</p>
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## Year 3 Addition



Objective/Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3digit numbers</p>	<p>Dienes or Numicon</p>  <p>Add together the ones first, then the tens.</p>   <p>Move to using place value counters</p>	 <p>Children move to drawing the counters using a 'tens and ones' frame.</p>	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>



<p>Column addition with regrouping</p>	 <p>Exchange ten ones for a ten. Model using Numicon and place value counters.</p> 	 <p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line.</p>	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ \hline 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$
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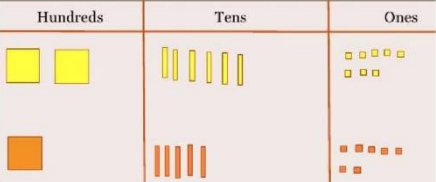
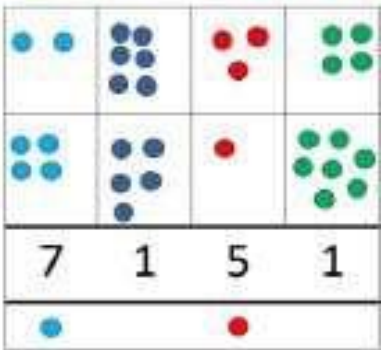
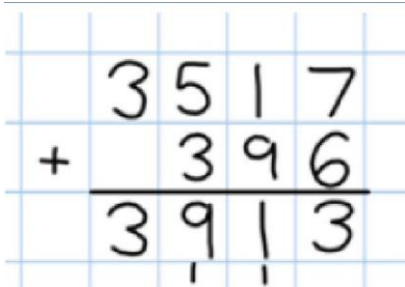
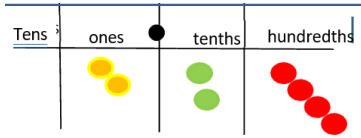
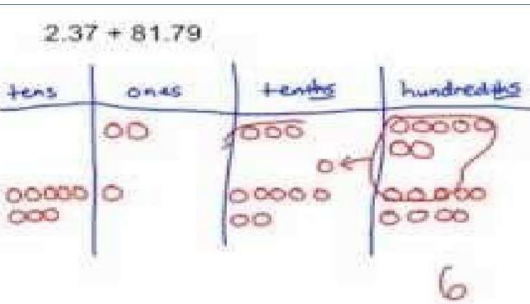
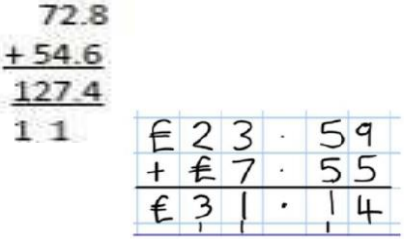
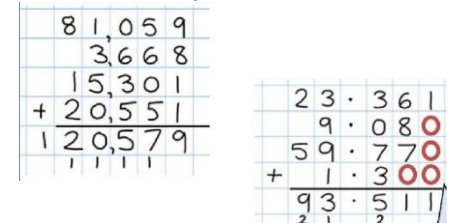
## Year 3

### Addition continued...

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Estimate the answers to questions and use inverse operations to check answers</p>	 <p>Estimating <math>98 + 17 = ?</math></p> <p><math>100 + 20 = 120</math></p>	<p>Use number lines to illustrate estimation</p> 	<p>Building up known facts and using them to illustrate the inverse and to check answers:</p> $\begin{array}{ll} 98 + 18 = 116 & 116 - 18 = 98 \\ 18 + 98 = 116 & 116 - 98 = 18 \end{array}$


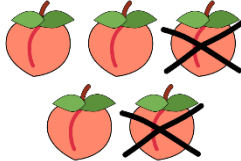






# Years 4, 5 and 6

## Addition

Objective/Strategy	Concrete	Pictorial	Abstract
<b>Year 4/5/6</b> Estimate and use inverse operations to check answers to a calculation	<b>As per Year 3</b>		
<b>Year 4</b> add numbers with up to 4 digits	 <p>Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p>	 <p>Draw representations using place value grid.</p>	 <p>Continue from previous work to carry hundreds as well as tens. Relate to money and measures.</p>
<b>Year 5</b> add numbers with more than 4 digits  add decimals with 2 decimal places, including money	<b>As per Year 4</b> 		
<b>Year 6</b> add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points	<b>As per Year 5</b>	<b>As per Year 5</b>	Insert zeros for place holders. 

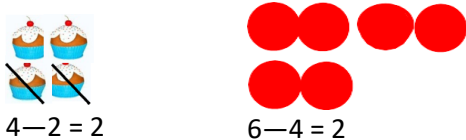
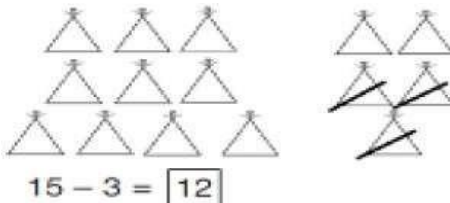

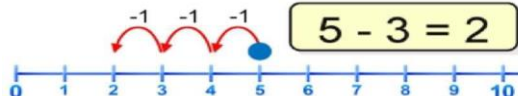
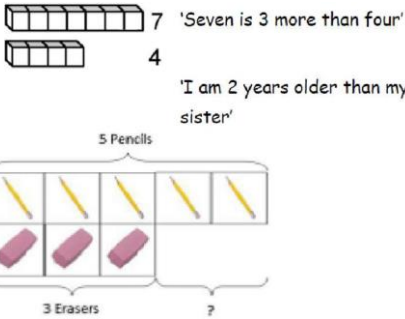
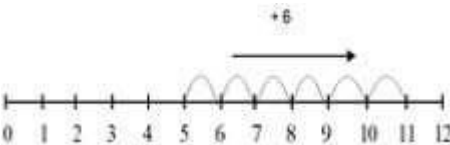
# Reception

## Subtraction

Objective/Strategy	Concrete	Pictorial	Abstract
Find 'one less than' a number	 <p>Use cubes to count out a given number. 'Take one away' and count to find 'how many left'.</p>	  <p>Use pictures to 'take one away'. Count to find out 'how many left'.</p>	 $3 - 1 = \underline{\quad}$ <p>Use concrete and pictorial examples to support moving into the abstract.</p> 
<p>To explore the composition of numbers to 10.</p> <p>Automatically recall number bonds for numbers 0–5 and some to 10.</p>	  <p>Use a feely bag to 'hide' a number of cubes. "I have 5 cubes altogether. I can see 2 cubes. How many have I hidden in my bag?"</p>  <p>Throw beanbags into a hoop. How many went in? How many are left outside?</p>		<p><b><i>Emphasis should be on the introduction and building up of subject-specific vocabulary through practical work.</i></b></p>

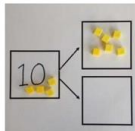
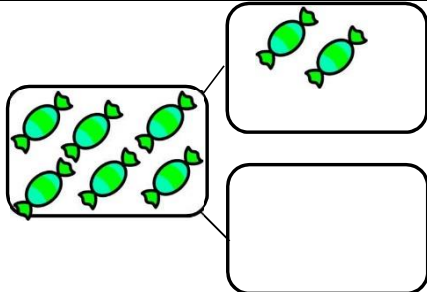
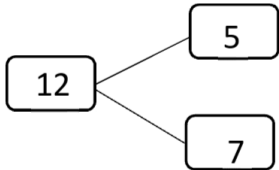
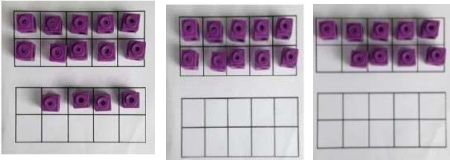
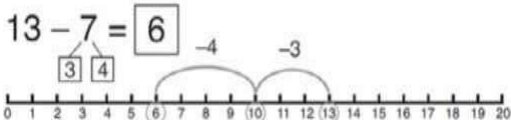
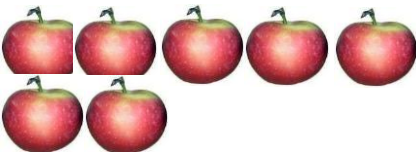

# Year 1

## Subtraction

Objective/Strategy	Concrete	Pictorial	Abstract
Taking away ones	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p><math>4 - 2 = 2</math>      <math>6 - 4 = 2</math></p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p><math>15 - 3 = 12</math></p>	<p><math>7 - 4 = 3</math></p> <p><math>16 - 9 = 7</math></p>
Counting back	 <p>Move objects away from the group, counting backwards.</p> <p>Move the beads along the bead string as you count backwards.</p>	 <p><math>5 - 3 = 2</math></p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>
Find the Difference	<p>Compare objects and amounts</p>  <p>Lay objects to represent bar model</p>	<p>Count on using a number line to find the difference.</p> 	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?</p>

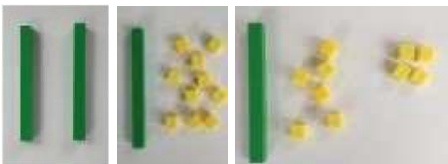


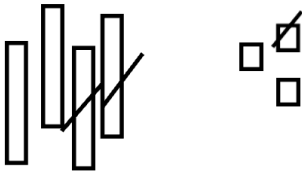
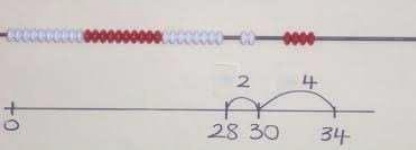
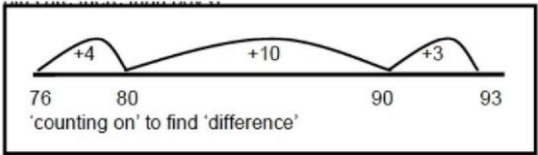
# Year 1

## Subtraction continued...



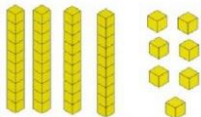
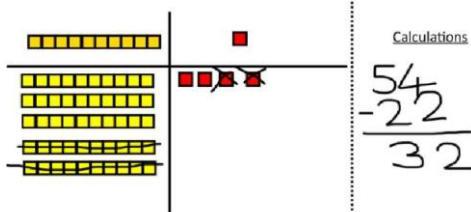
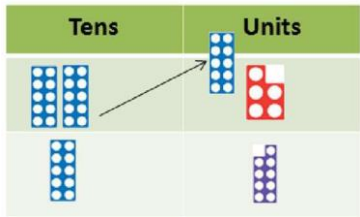
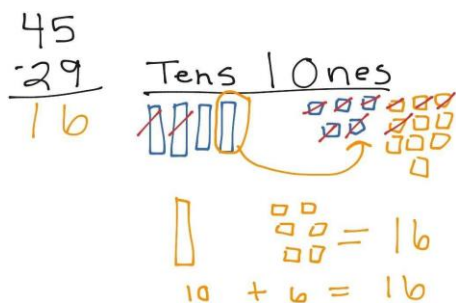
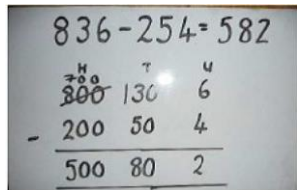
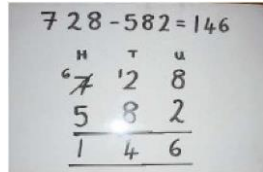
Objective/Strategy	Concrete	Pictorial	Abstract		
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Include subtracting zero</p> <p><i>Part, Part Whole model</i></p>	<div></div> <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the arts, what s the other part?</p> <p><math>10-6=4</math></p>	<div></div> <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p> <div></div> <p>Include missing number problems: <math>12-?=5</math> <math>7=12-?</math></p>		
<p>Make 10</p>	<div><p><math>14-9</math></p></div> <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<div><p><math>13-7=6</math></p></div> <p><math>13-7</math></p> <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p><math>16-8</math></p> <p>How many do we take off first to get to 10? How many left to take off?</p>		
<p>Bar model</p> <p>Including the inverse operations.</p>	<div></div> <p><math>5-2=3</math></p>	<div></div>	<div><table><tr><td>8</td><td>2</td></tr></table></div> <p><math>10=8+2</math> <math>10=2+8</math> <math>10-2=8</math> <math>10-8=2</math></p>	8	2
8	2				

## Year 2

### Subtraction

Objective/Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 $20 - 4 =$	$20 - 4 = 16$
Partitioning to subtract without regrouping  <i>'Friendly numbers'</i>	$34 - 13 = 21$  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	Children draw representations of Dienes and cross off.   $43 - 21 = 22$	$43 - 21 = 22$
'Make ten' strategy  <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds</i>	 $34 - 28$ <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	$93 - 76 = 17$

## Year 3 Subtraction

Objective/Strategy	Concrete	Pictorial	Abstract
Subtract numbers mentally, including:  three-digit number + ones  three-digit number + tens  three-digit number + hundreds			Vary the position of the answer and question.  Expose children to missing number questions and vary the missing part of the calculation. $678 = ? - 1$ $688 - 10 = ?$ $678 = ? - 100$
Column subtraction without regrouping  <i>'Friendly numbers'</i>	 $47 - 32$  Use Base 10 or Numicon to model	 Draw representations to support understanding	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$  Intermediate step may be needed to lead to clear subtraction understanding
Column subtraction with regrouping	  Begin with base 10 or Numicon. Move to PV counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange	  Children may draw base ten or PV counters and cross off	 Begin by partitioning into PV columns   Then move on to formal method




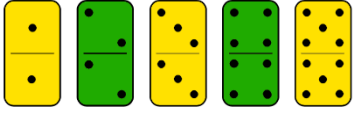

# Years 4, 5 and 6

## Subtraction

Objective/Strategy	Concrete	Pictorial	Abstract
<b>Year 4</b> subtract tens and ones with up to 4 digits  introduce decimal subtraction through context of money	<p>234 - 179</p> <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw PV counters and show their exchange—<b>see Year 3</b></p>	<p>Use the phrase 'take and make' for exchange</p>
<b>Year 5</b> subtract with at least 4 digits, including money and measures  subtract with decimal values, including mixtures of integers and decimals and aligning the decimal point (up to 3 decimal places)	<p><b>As per Year 4</b></p>	<p>Children to draw PV counters and show their exchange—<b>see Year 3</b></p>	<p>Use zeros for placeholder</p>
<b>Year 6</b> Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal places)	<p><b>As per Year 4</b></p>	<p>Children to draw PV counters and show their exchange—<b>see Year 3</b></p>	

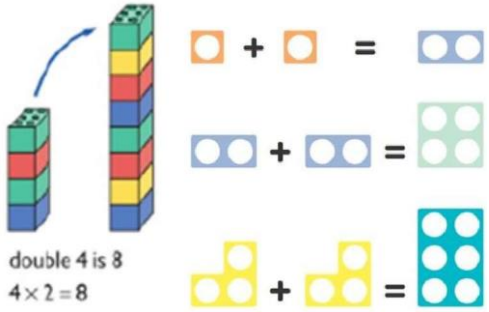

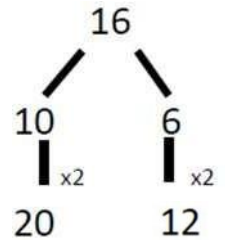
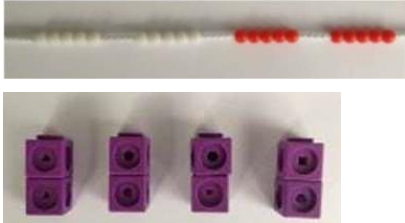
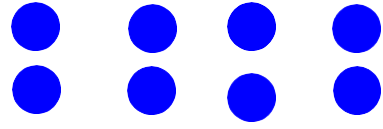
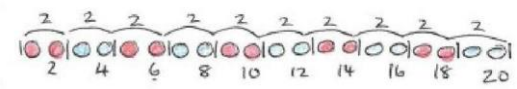
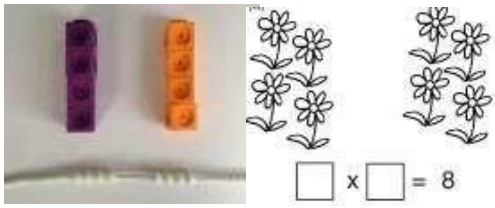



# Reception Multiplication

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Subitise</p> <p>Compare numbers</p>	 <p>Roll a pair of dice to find doubles</p>  <p>Match pairs of Dominoes to find doubles</p>	 <p>Use pictures to find doubles by drawing the same again</p>	<p>Emphasis should be on the introduction and building up of subject-specific vocabulary through practical work.</p>

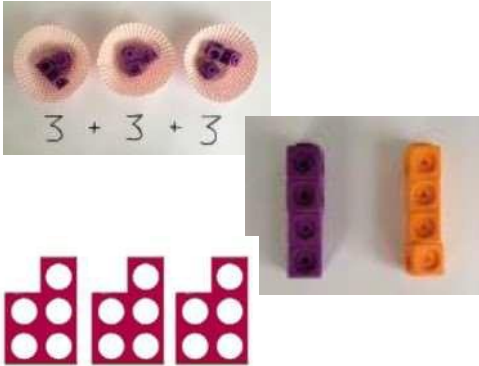
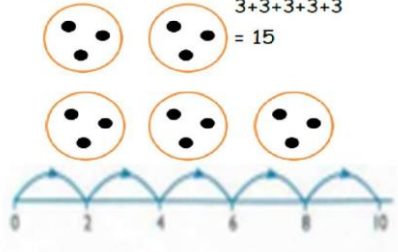

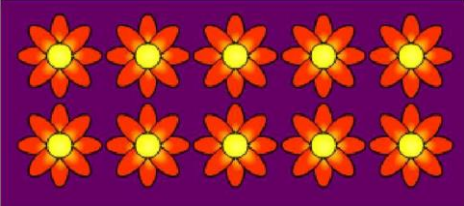
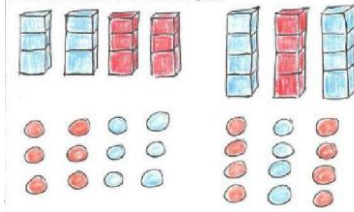
# Year 1

## Multiplication

Objective/Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 <math>4 \times 2 = 8</math></p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together</p>  <p><math>20 + 12 = 32</math></p>
Counting in multiples (2s, 5s, 10s)	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers. 2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>
Making equal groups and counting the total	 <p>Use manipulatives to create equal groups.</p> <p><math>\square \times \square = 8</math></p>	<p>Draw  to show <math>2 \times 3 = 6</math></p> <p>Draw and make representations</p>	<p><math>2 \times 4 = 8</math></p>

# Year 1

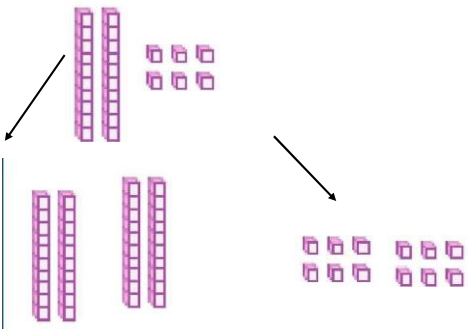


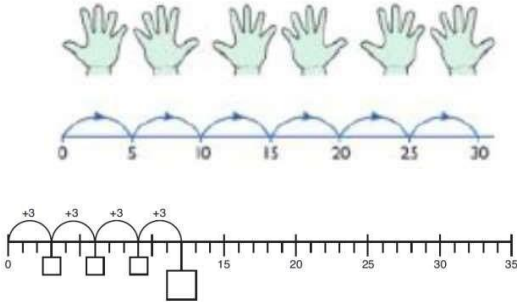
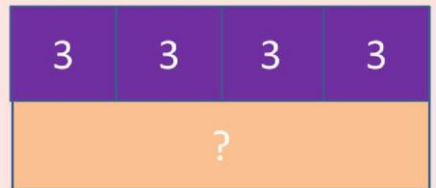
## Multiplication continued...

Objective/Strategy	Concrete	Pictorial	Abstract
Repeated addition	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve problems</p> <p>prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> <p><math>3+3+3+3+3 = 15</math></p> 	<p>Write addition sentences to describe objects and pictures</p>  <p><math>2 + 2 + 2 + 2 + 2 = 10</math></p>
Understanding arrays	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p> 	<p><math>3 \times 2 = 6</math></p> <p><math>2 \times 5 = 10</math></p>

# Year 2




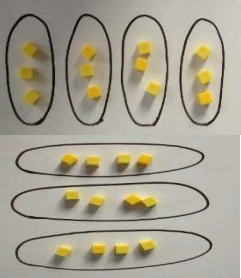
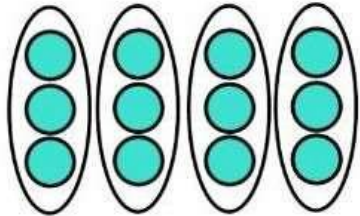
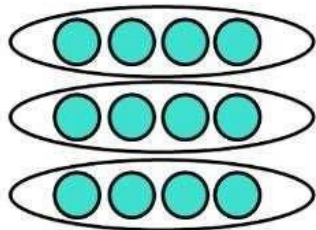

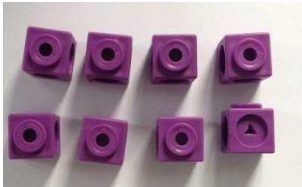
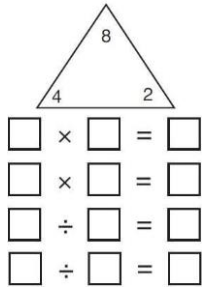
## Multiplication

Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times tables.

Objective/Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Model doubling using dienes and PV counters.</p>  <p><math>40 + 12 = 52</math></p>	<p>Draw pictures and representations to show how to double numbers</p>	<p>As per Year 1</p>
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p>  <p><math>5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40</math></p> <p>Use bar models.</p> 	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p>  	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30</p> <p><math>4 \times 3 = \square</math></p>

## Year 2

### Multiplication continued...

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters, cubes or Numicon.</p>    <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity.</p>  	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$ <p><b><math>12 = 3 \times 4</math></b>  <b><math>12 = 4 \times 3</math></b></p>
<p>Using the Inverse</p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p>			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ <p>Show all 8 related fact family sentences.</p>

# Year 3

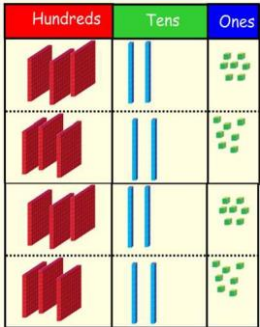
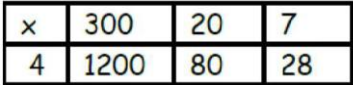
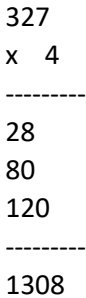
## Multiplication

Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables.

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Grid method, progressing to the formal method</p> <p>Multiply 2-digit numbers by 1-digit numbers</p>	<p>Show the links with arrays to first introduce the grid method.</p> <div><div><div><div>10</div><div>3</div></div><div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div></div></div><div>4 rows of 10 4 rows of 3</div></div><p>Move onto base ten to move towards a more compact method.</p><div><div><div><div>T</div><div>U</div></div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div></div></div><div>4 rows of 13</div></div><p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p><div><div><div><div>100</div><div>10</div><div>1</div></div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div></div></div><div>Calculations 4 x 126</div></div><div><div><div><div>100</div><div>10</div><div>1</div></div><div><div><div>100</div><div>100</div><div>100</div><div>100</div></div><div><div><div>10</div><div>10</div><div>10</div><div>10</div><div>10</div><div>10</div><div>10</div><div>10</div></div><div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div></div></div><div>Calculations 4 x 126</div></div><p>Fill each row with 126.</p><p>Add up each column, starting with the ones making any exchanges needed Then you have your answer.</p></div></div></div></div></div></div></div></div>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> <div><div><div>24 x 3 = 72</div><div><div><div>X</div><div>20</div><div>4</div></div><div><div>3</div><div>00</div><div>00</div><div>00</div><div>60</div></div><div><div>0000</div><div>0000</div><div>0000</div><div>12</div><div>60</div><div>+ 12</div><div>72</div></div></div></div></div> <p>Bar models are used to explore missing numbers</p> <div><div><div>4 x <div></div> = 20</div><div><div><div>20</div><div>4</div></div></div></div></div>	<p>Start with multiplying by one-digit numbers and showing the clear addition alongside the grid.</p> <div><div><div><div>X</div><div>30</div><div>5</div></div><div><div>7</div><div>210</div><div>35</div></div></div></div> <div><div>210 + 35 = 245</div></div> <p>Move forward to the formal written method:</p> <div><div><div>35</div><div>X 7</div><div><div>245</div><div>3</div></div></div></div>

# Years 4, 5 and 6

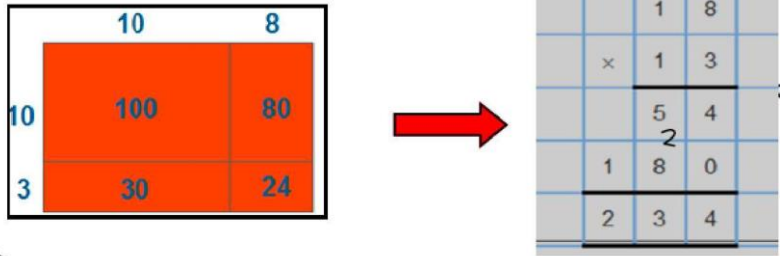
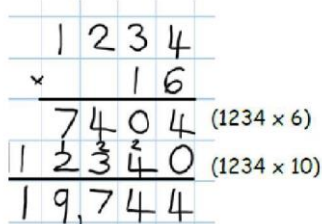
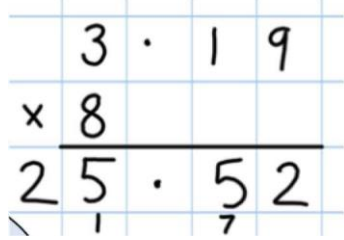
## Multiplication

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Grid method to recap from Year 3 for 2-digit x 1-digit</p> <p>Move to multiplying 3-digit numbers by 1-digit</p> <p>Year 4 expectation</p>	<p><b>As per Year 3</b></p>	<p><b>As per Year 3</b></p>	<p><b>As per Year 3</b></p>
<p>Column Multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This is initially done where there is no regrouping, such as:</p> <p><math>321 \times 2 = 642</math></p>  <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding expanded multiplication is modelled alongside.</p>		






# Years 4, 5 and 6

## Multiplication

Objective/Strategy	Concrete	Pictorial	Abstract
Column multiplication (Long multiplication)	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 <p>Continue to use bar modelling to support problem solving</p>	
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the Units column. Line up the decimal points in the question and the answer. 

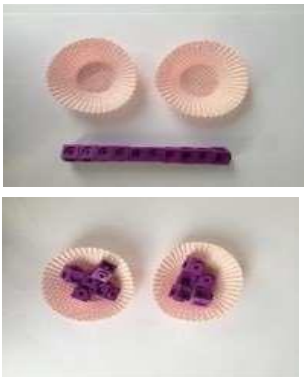
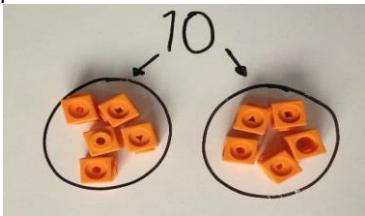

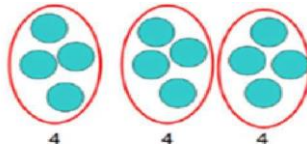
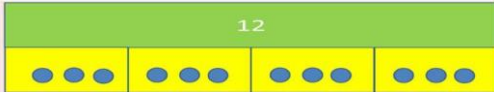
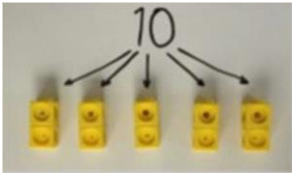
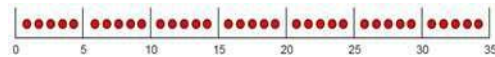
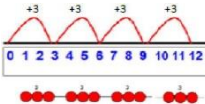



# Reception Division

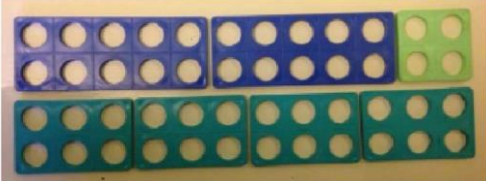
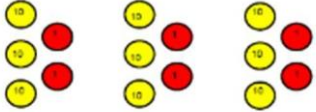
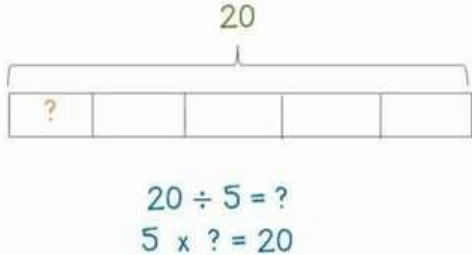
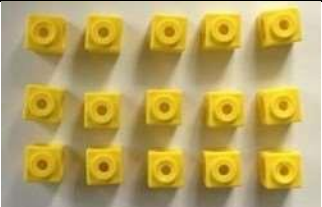
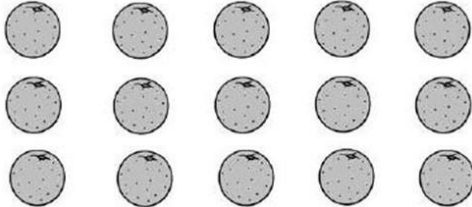
Objective/Strategy	Concrete	Pictorial	Abstract
Explore the composition of numbers to 10	 <p>Share the cubes equally between 2, e.g. "One for you, one for me".</p>  <p>Show the children some unequal groups of objects. Is this fair or unfair? Are the groups the same or different?</p>	<p>There are 6 cakes. Can you share them? Try saying "one for me, one for you"</p>  <p>Use pictures to share</p>	<p><i>Emphasis should be on the introduction and building up of subject-specific vocabulary through practical work.</i></p>

# Year 1

## Division

Objective/Strategy	Concrete	Pictorial	Abstract
Division as sharing <i>(Use Gordon ITPs for modelling)</i>	 <p>I have 10 cubes; can you share them equally in 2 groups?</p> 	<p>Children use pictures or shapes to share quantities:</p>  <p>8 shared between 2 is 4.</p> <p>Sharing:</p>  <p>12 shared between 3 is 4</p> <p>Children use bar modelling to show and support understanding.</p>  <p>12 ÷ 4 = 3</p>	<p>12 shared between 3 is 4.</p> <p>12 ÷ 3 = 4</p>
Division as grouping	<p>Divide quantities into equal groups</p>  <p>Use cubes, counters, objects or place value counters to aid understanding.</p> 	<p>Use number lines for grouping</p>  <p>12 ÷ 3 = 4</p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p>20 ÷ 5 = ? 5 x ? = 20</p>	<p>28 ÷ 7 = 4</p> <p>Divide 28 into 7 groups. How many are in each group?</p>

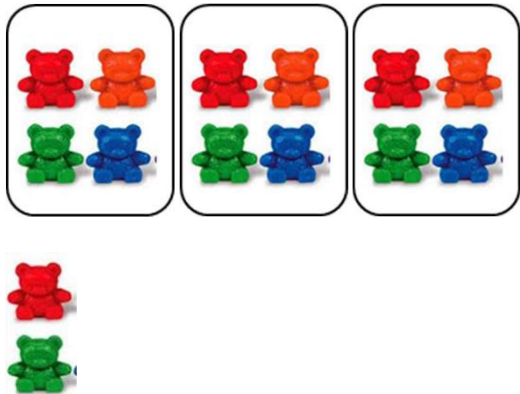
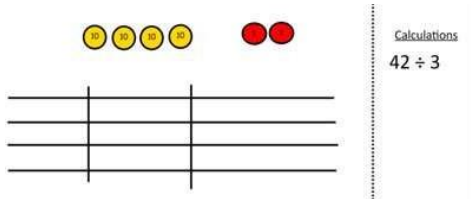
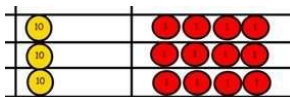
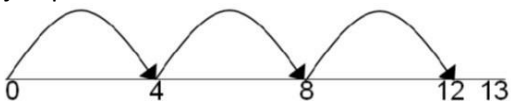
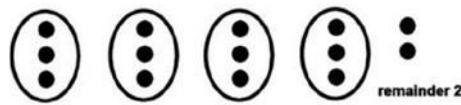
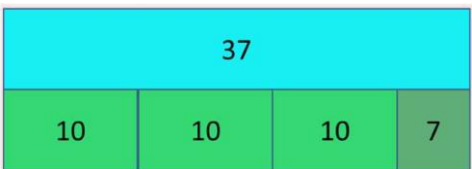
## Year 2 Division

Objective/Strategy	Concrete	Pictorial	Abstract
Division as grouping	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p> 	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
Division with arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> $15 \div 3 = 5 \qquad 5 \times 3 = 15$ $15 \div 5 = 3 \qquad 3 \times 5 = 15$	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences:</p>  $15 \div 3 = 5$	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28 \qquad 28 = 7 \times 4$ $4 \times 7 = 28 \qquad 28 = 4 \times 7$ $28 \div 7 = 4 \qquad 4 = 28 \div 7$ $28 \div 4 = 7 \qquad 7 = 28 \div 4$

# Year 3

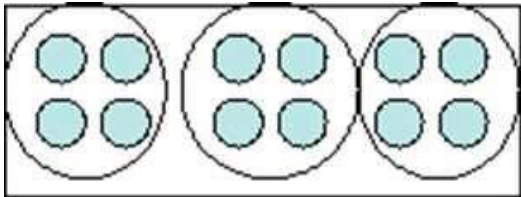
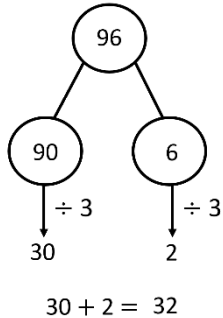
## Division

(Greater Depth Y2)

Objective/Strategy	Concrete	Pictorial	Abstract
Division with remainders	<p>Divide objects between groups and see how much is left over:</p>  <p><math>14 \div 3 = 4</math> with 2 left over</p> <p><math>96 \div 3</math></p>  <p>Use place value counters to divide using the bus stop method alongside:</p> <p><math>42 \div 3 =</math></p>  <p>We look how much in 1 group, so the answer is 14</p>	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder:</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder:</p>  <p>Use bar models to show division with remainders:</p> 	<p>Complete written divisions and show the remainder using r.</p> <p><math>29 \div 8 = 3 \text{ REMAINDER } 5</math></p> <p>↑   ↑   ↑   ↑ dividend   divisor   quotient   remainder</p>

# Years 4, 5 and 6

## Division

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Divide at least 3-digit numbers by a 1-digit number</p> <p>Short Division</p>	<p><b>As per Year 3</b></p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently</p> <p><math>96 \div 3 =</math></p> 	<p>Begin with division that divide equally with no remainder</p> $\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$ <p>Move onto division with a remainder</p> $\begin{array}{r} 0663r5 \\ 8 \overline{) 5309} \end{array}$ <p>Finally move into decimal places to divide the total accurately</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$

# Year 6

## Division

Objective/Strategy	Abstract
Long division	<div><div>Long division: a step-by-step guide</div><p>Long division is set out in the following way.</p><div><div><div><div>15</div><div>3640</div></div><div><div>2</div><div>15</div><div>3640</div><div>-30</div><div>6</div></div></div><div><div>15</div><div>3640</div></div><div><div>24</div><div>15</div><div>3640</div><div>-30</div><div>64</div><div>-60</div><div>4</div></div></div><div><div>15</div><div>3640</div></div><div><div>242</div><div>15</div><div>3640</div><div>-30</div><div>64</div><div>-60</div><div>40</div><div>-30</div><div>10</div></div></div> <div><div>15 into 3 doesn't go, so look at the next digit.</div><div>15 goes into 36 two times, so put a 2 above the 6. <math>15 \times 2 = 30</math></div><div>Take that 30 away from the 36 to get your remainder. <math>36 - 30 = 6</math></div><div>Next, carry the 4 down to make 64.</div><div>15 goes into 64 four times, so put a 4 above the 4. <math>15 \times 4 = 60</math></div><div>Take 60 from the 64 to get your remainder. <math>64 - 60 = 4</math></div><div>Carry the 0 down to make 40.</div><div>15 goes into 40 two times, so put a 2 above the 0. <math>15 \times 2 = 30</math></div><div>Take 30 from the 40 to get your remainder. <math>40 - 30 = 10</math></div></div> <div><div><div>34</div><div>1598</div></div><div><div>4</div><div>34</div><div>1598</div><div>-136</div><div>23</div></div></div> <div><div>34</div><div>1598</div></div> <div><div>47</div><div>34</div><div>1598</div><div>-136</div><div>238</div></div>

At the side of the calculation, do jottings for multiples of 34.

34 doesn't go into 15, so look for the closest multiple to 159.

$4 \times 34$  is 136 so subtract this from 159, leaving 23.

Move the 8 that is left in the dividend down to make 238 left.

Use the jottings created already to find the closest multiple of 34 to 238. In this case it is  $7 \times 34$ , which makes a quotient of 47.

If there is a remainder this can be recorded as a remainder, or fraction, or the method can be extended to incorporate decimals as necessary.